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ECONOMIC-MATHEMATICAL MODEL FOR OPTIMAL DISTRIBUTION OF SERVICE STATION

The paper studies a particular problem of optimal coverage or partitioning of a set of consumers into service areas without restrictions on the power of service stations, locating the coordinates of subset centers. We analyze existing formulations of related problems and methods for their solution. We construct an economic-mathematical model of the problem, develop an algorithm for its solution. We propose an efficiency criterion of the partition for service areas which is defined as the coefficient of uneven load on consumers. We provide examples of practical applications for the developed model and algorithm. Solving such problems requires additional research, including collecting and statistically processing additional information about possible ways of consumer movements and service station locations. An analysis of the infrastructure of a populated area or region will allow to identify bottlenecks in the functioning of various organizations, elimination of which will lead to an increase in the quality of life of the population.

Key words: economic-mathematical model, optimal partitioning, service station, algorithm criterion, partitioning efficiency, characteristic function.

Problem statement. Solving a task of distributing service stations involves a subtask of partitioning a set of consumers into subsets which ensures that consumers as located close to the service objects. After that it's essential to determine the optimal location of the service station for each subset. Such a task is usually completed in several stages. As a result, it is necessary to repeatedly combine the methods of partitioning with the methods of placement. In addition to that, it is advisable to formulate a criteria for assessing the effectiveness of service station placement and partitioning into the subsets. Such an assessment is required for populated areas or regions that are actively developing, with current infrastructure which provided sufficiently satisfactory operation for some time ago, but now became ineffective, which is the reason for assessment of its current state and development of recommendations for its modernization.

Analysis of recent research and publications. Economic problems of service station placement can be formulated as the problem of the optimum set coverage.

Such problems include, for example, the problem of placing the fixed number of cellular telephony stations and determining the minimum range of the effective communication; the problem of identifying the required number of the cellular station with fixed effective radius and placing said stations; the problem of determining the minimum required radius of water spread of a watering installation along with placing a predetermined number of these installations in the area of irrigation; the task of building a network of public service stations; and many others.

Problem formulations and solutions use the following definitions.

Ω – bounded compact set in Euclidean space E_n.

c-sphere with radius R and center τ_i in E_n – a set defined by $B(\tau_i,R) = \{x \in E_n: c(x,\tau_i) \le R\}$, where $c(x,\tau_i)$ is a certain quasimetric [9].

A collection of centers $\tau_1,\dots,\tau_N$ defines a sphere coverage of set Ω with radius R if $\Omega \subseteq \bigcup_{i=1}^{N} B(\tau_i,R)$.

Radius R of coverage of the set Ω defined by centers $\tau_1,\dots,\tau_N$ (a vector τ^N) has form

$$R(\tau^N) = \sup_{x\in\Omega} \min_{i=1,\dots,N} c(x,\tau_i),$$

where $\tau^N \in \underbrace{E_n \times \dots \times E_n}_{N}$ (or $\tau^N \in \underbrace{\Omega \times \dots \times \Omega}_{N}$).

In order to achieve an optimal coverage one needs to determine the coverage radius $R(\tau^N) = \inf_{\tau^N} \sup_{x\in\Omega} \min_{i=1,\dots,N} c(x,\tau_i)$ and the vector $\tau_*^N = (\tau_1^*,\dots,\tau_N^*)$, which yields the radius value equal to the lowest bound [9, 12].

Research [6] formulates several coverage problems using continuous c-spheres. Such problems of optimal set coverage using a finite system of subsets have been actively researched since the 1960s and maintain continuous scientific and practical attention since then [2, 3, 6 − 12].

For example, V. Brusov and S. Piyavsky in [8] state several of the most popular formulations of the optimal set coverage problem. They study the first two problems from [6] and prove that a solution always for these problems. [8] also provides a generic algorithm for solving the problem 2, and formulates the requirements for a local minimum in the case of a real axis is covered by a set of centers which is a closed bounded region of the euclidean space.

Research [2] dissects an algorithm for solving the problem 2 in the case of two closed bounded regions of a plane X and Y are considered as the covered area and the area of possible center locations. The distance function $\rho(x,y)$ for points x and y is continuous, non-negative, and convex in X for all $y \in Y$. The problem considered in [2] is a coverage problem for the set X *n* using convex areas of minimal radius with centers in $y_i \subset Y$, $i = 1,...n$. The following notation is then used. Let $A(\bar{y})$ be the maximum distance in the Dirichlet region of $\bar{y}$. The center $\bar{y}_i$ of the Dirichlet region is unimprovable if there is no point $\bar{y}_i$ which is infinitely close to $\tilde{y}_i$ and has $A(\tilde{y}_i) < A(\bar{y}_i)$. Otherwise the center $\bar{y}_i$ is considered an improvable one.

The main idea of an algorithm proposed in [2] is to construct a Dirichlet-Voronoi tessellation for the region X using an initial center vector $\left(y_1^0,...,y_n^0\right)$. Then an improvable center y_i^0, $i = 1,...n$, n having a maximal value of A is selected and replaced with an unimprovable center y_i^1. The algorithm loops until all centers become unimprovable. As each iteration improves position of only one center, the algorithm does not show magnificent speed. Furthermore, the implementation of the algorithm is highly dependent of the form of the region to be covered. However, the proposed algorithm is proved to be convergent.

Research [11] considers and proposes solutions of a coverage problem for a compact connected set in E^n. The functional of the coverage problem is mathematically defined as a penalty function weighted by some statistical distribution. Thus the problem is find m points $y_1,...,y_m \in Y$, which minimize the following functional

$$\sum_{i=1}^{m} \int_X f_i(x, y_1,..., y_m) q(x, y_i) dF(x)$$

where X is a compact connected set in E^n; $Y \subseteq X$; *F(x)* is the probability distribution function defined for X; $f_i(x, y_1,..., y_m)$, $i = 1,...,m$, *m* are some functions such that $\sum_{i=1}^{m} f_i(x, y_1,..., y_m) = 1$; *q(x, y)* is the penalty function which is equal to zero for *x=y* and is positive otherwise but the penalty function is not required to satisfy the triangle inequality.

The algorithm proposed in [11] takes into account the weight of the points in the region and describes a method to avoid the local minimum traps.

Research [12] views the continuous set coverage problem as a problem of quasidifferential optimization in R^n and provides an algorithm for the case of R^2. They also explore finite spherical coverage of compact sets, study the properties of this problem, and propose an algorithm for the case of bounded problem of finite sphere coverage of compacts sets in R^2. The algorithm can find stationary points, but it has restrictions on the form of the set to be covered thus it is impossible to build generic set coverage software with it.

A comparative analysis of the problem of minimal radius of set coverage versus the problem of optimal set tessellation can be found in [7]. They show that optimal tessellation algorithms are not guaranteed to find a global solution of the problem of optimal coverage using spheres with the smallest radius for arbitrary cases. However, sufficiently developed and advanced algorithms of optimal tessellation are deemed to be justified and appropriate for the problem of optimal set coverage.

Research [10] proposes a method for covering a polygonal region P with the smallest number of circles with given radius. A special kind of function is used to evaluate the coverage:

$$\Phi(v,u) = \Phi(x, y, u) = \min\{\varphi_1(v,u_1),...,\varphi_k(v,u_k)\},$$

where $\varphi_i(v,u_i) = (x - x_i)^2 + (y - y_i)^2 - r^2$, $i = 1,2,...,k$, $v, u_i \in P$, $u = (u_1,...,u_k)$ is the set of center points.

If there exists $u = u^*$ such as $F(v) = \max_{v \in P} \Phi(v,u^*) \leq 0$, then it is the solution of the problem. The paper provides a generic algorithm for an ancillary problem. Each step consists of constructing a Dirichlet-Voronoi tessellation for the current center points and verifying that $\max_{v \in B} F(v) = 0$. If the condition is not true then u is minimized using the method of available directions and the algorithm takes another iteration. The resulting solution may be a local minimum, non-strict local minimum, or a saddle point. Determining the kind of the point requires solving another ancillary optimization problem which implies additional computations.

Research [6] justifies a method for solving the problem of optimal c-spherical coverage of a convex set in E^n with an arbitrarily defined metric. It is based on a generic gradient descent method with space extruded towards the difference between two consecutive generalized gradients. They develop a corresponding algorithm and study its efficiency. The algorithm can be used for set coverage problems when the sets are convex, discrete, and contain differently weighted elements.

Formulation of aims of article. The purpose of this work is to study a particular problem of optimal coverage or partitioning of a set of consumers into service areas without restrictions on the power of service stations, yielding the coordinates of the subset centers. It is necessary to construct an economic-mathematical model of the problem and to develop an algorithm for its solution. We also aim to establish a criterion for efficiency of the partitioning of service areas. The economic problems of the service station placement can be formulated as problems of optimal set coverage (partitioning) [5]. Such problems include, for example, the problem of placing a fixed number of cell phones and determining the minimum range of effective communication; the problem of determining the required number of cellular stations with a fixed effective radius and placement of these stations; the problem of determining the minimum required radius of water distribution in an irrigation installation, along with the placement of a predetermined number of these plants in the irrigation area; the task of building a network of public service stations; and many others.

Presenting of the main material. We will assume that service stations and their consumers can be located anywhere in the region, the number of consumers at each point may be different.

Let there be a delimited closed area $S \subseteq E_2$. The consumers are located at points $(x,y) \in S$ of the area, with quantity of the consumers or their service requirements defined by a non-negative function $f(x,y) \geq 0$. Velocity of consumers is defined by a function $v(x,y)$ which depends on the terrain, quality of the road surface, fuel expenses, season, type of transport, etc. Furthermore, there is a finite number ($m>1$) of service stations with initially unknown locations $\varsigma_i = (\varsigma_i^x, \varsigma_i^y)$, $i=\overline{1,m}$. Then for each point $(x,y) \in S$ the minimal time required to move from (x,y) to the station ς_i, $i=\overline{1,m}$ can be defined as

$$t_i(x,y) = \inf_{p_i(\mathrm{x,y}) \in \mathrm{P}} \int_{p_i(\mathrm{x,y})} \frac{dp_i(\mathrm{x,y})}{v(x,y)},\ i=\overline{1,m}, \tag{1}$$

where $p_i(\mathrm{x,y}) \in \mathrm{P}_i$, $i=\overline{1,m}$, P_i is the set of all possible routes connecting points (x,y) and ς_i.

We need to define locations of the service stations $\varsigma_i, i=\overline{1,m}$ and determine an optimal partitioning of the consumers into m subsets $S_1^*,...,S_m^*$ by defining for each consumer at $(x,y) \in S$ which station $\varsigma_i^* \in S_i^*$, $i=\overline{1,m}$, they should use for service in order for the total time spent for commute to the service station to be minimum, that is:

$$(((\mathrm{x,y}) \in S_1,...,(\mathrm{x,y}) \in \mathrm{S_m}),(\varsigma_1,...,\varsigma_m))$$

$$\sum_{i=1}^{m} \int_{S_i} f(x,y) t_i(x,y) dxdy \to \min_{((S_1,...,\mathrm{S_m}),(\varsigma_1,...,\varsigma_m))}, \tag{2}$$

(in case of continuous distribution of consumers), or

$$\sum_{i=1}^{m} \sum_{(x,y)\in S_i} f(x,y) t_i(x,y) \to \min_{(((\mathrm{x,y})\in S_1,...,(\mathrm{x,y})\in \mathrm{S_m}),(\varsigma_1,...,\varsigma_m))}, \tag{3}$$

(in case of the number of consumers ($n>m$) is limited).

It should be noted that a solution to the problem (3) can be obtained from problem (2) with $f(x,y)=0$ for each point $(x,y) \in S$ which is not a location of a consumer in the region S.

The problem (2) can be solved using the method proposed in [5]. This involves transforming (2) into an equivalent form using characteristic functions $\mu_i(x,y)$ of sets S_i, $i=\overline{1,m}$:

$$\mu_i(x,y)=\begin{cases}1, & якщо \quad (x,y)\in S_i \\ 0, & якщо \quad (x,y)\notin S_i\end{cases}. \tag{4}$$

With a pair of values $(\mu_*(x,y),\varsigma_*)$ such that

$$\mathrm{F}(\mu_*(x,y),\varsigma_*)=\min_{(\mu(x,y),\varsigma)\in\mathrm{M}\times S^m}\mathrm{F}(\mu(x,y),\varsigma), \tag{5}$$

where

$$\mathrm{F}(\mu(.),\varsigma)=\int_S\sum_{i=1}^m f(x,y)t_i(x,y)\mu_i(x,y)dxdy \tag{6}$$

$$\mathrm{M}=\{\mu(x,y)=(\mu_1(x,y),...,\mu_m(x,y)):$$

$$\sum_{i=1}^m\mu_i(x,y)=1,\quad \text{м.в.}\quad (x,y)\in S, \tag{7}$$

$$\mu_i(x,y)=0 \quad або \quad 1 \quad \text{м.в.}\quad (x,y)\in S,\quad i=\overline{1,m}\},$$

$$\varsigma=(\varsigma_1,...,\varsigma_m)\in\underbrace{\mathrm{S}\times...\times\mathrm{S}}_{m}=S^m.$$

Let us describe the solution for this problem.

It is obvious from [5] that

$$\mathrm{F}(\mu_*(.),\varsigma_*)=\min_{(\mu\,\varsigma)\in\mathrm{M}\times S^m}\mathrm{F}(\mu(.),\varsigma)=\min_{\varsigma\in S^m}\ (\min_{\mu(.)\in\mathrm{M}}\mathrm{F}(\mu(.),\varsigma)). \tag{8}$$

As in [5], an optimal solution for the internal subtask in (8) is achieved when $\varsigma\in S^m$ is fixed for a vector function $\mu_*(x,y)=(\mu_{*1}(x,y),...,\mu_{*\mathrm{m}}(x,y))$ with i-th component defined as:

$$\mu_{*_i}(x,y)=\begin{cases}1, & якщо \quad (x,y)\in S_{*_i} \\ 0, & якщо \quad (x,y)\notin S_{*_i}\end{cases},\ i=\overline{1,m}, \tag{9}$$

where

$$S_{*\mathrm{i}}=\{(\mathrm{x},\mathrm{y}):\quad t_i(x,y)=\min_{l=1,m}t_l(x,y)\}. \tag{10}$$

Let us define

$$\mathrm{P}(\varsigma)=\min_{\mu(.)\in\mathrm{M}}\mathrm{F}(\mu(.),\varsigma),\ \varsigma\in S^m. \tag{11}$$

Following [5], the external subtask in (8) is defined like this:

$$\mathrm{P}(\varsigma)=\int_S\min_{l=1,m}f(x,y)t_i(x,y)dxdy\to\min, \tag{12}$$

$$\varsigma\in S^m.$$

Let us describe the algorithm for solving this problem.

The basis of the algorithm is formed by equations (9-10) and a certain variation on generalized gradient descent with space expansion in the direction of the difference between two consecutive generalized gradients (so called r-algorithm), which was developed for finding local minimums of an undifferentiatable multi-extremum function $P(\varsigma)$ from the problem (12).

Detailed descriptions of different variations of the r-algorithm can be found in [13].

We use the H-form of the r-algorithm [13] (H_k is a symmetric matrix where $H_k = B_k B_k^T$) which results in the following iterative equation:

$$\varsigma^{k+1} = \varsigma^k - h_k \frac{H_{k+1} g_P(\varsigma^k)}{\sqrt{(H_{k+1} g_P(\varsigma^k), g_P(\varsigma^k))}}, \quad k = 0,1,2... \tag{13}$$

where

$$H_{k+1} = H_k + (1/\alpha_k^2 - 1)\frac{H_k \Delta_k \Delta_k^T H_k}{(H_k \Delta_k, \Delta_k)},$$

$$\Delta_k = g_P(\varsigma^k) - g_P(\varsigma^{k-1}).$$

Coefficient of space expansion is α_k=3. Iteration multiplier h_k is adaptively controlled in accordance with [13].

Inscribe area S into a parallelepiped Π with sides parallel to coordinate axes. Let $f(x,y)=0$ for $(x,y)\in \Pi \setminus S$. Cover parallelepiped Π with rectangular grid and let the initial guess be $\varsigma = \varsigma^{(0)}$. Compute value $\mu^{(0)}(x,y)$ for grid nodes using equations (9) with $\varsigma = \varsigma^{(0)}$, and the gradient value $g_P(\varsigma^0)$ with $\mu(x,y) = \mu^{(0)}(x,y)$, $\varsigma = \varsigma^{(0)}$, after which select the initial r-algorithm step $h_0>0$ and compute

$$\varsigma^1 = PR_\Pi \left(\varsigma^0 - h_0 \frac{H_1 g_P(\varsigma^0)}{\sqrt{(H_1 g_P(\varsigma^0), g_P(\varsigma^0))}} \right),$$

where PR_Π denotes projection onto Π.

Repeat r-algorithm iterations.

After k iterations we have certain values of $\varsigma^{(k)}$, $\mu^{(k)}(x,y)$ in grid nodes. Let's describe the ($k+1$)-th iteration.

1. Compute values $\mu^{(k)}(x,y)$ in grid nodes with $\varsigma = \varsigma^{(k)}$.
2. Determine the value of $g_P(\varsigma)$ with $\mu(x,y) = \mu^{(k)}(x,y)$, $\varsigma = \varsigma^{(k)}$.
3. Perform ($k+1$)-th iteration of the H-form of the r-algorithm [13] which has the following iterative equation:

$$\varsigma^{k+1} = PR_\Pi \left(\varsigma^k - h_k \frac{H_{k+1} g_P(\varsigma^k)}{\sqrt{(H_{k+1} g_P(\varsigma^k), g_P(\varsigma^k))}} \right).$$

4. If the following condition

$$\| \varsigma^{k+1} - \varsigma^k \| \le \varepsilon, \quad \varepsilon>0, \tag{14}$$

does not hold then proceed to ($k+2$)-th iteration. Otherwise go to step 5.

4. Let $\varsigma_* = \varsigma^{k+1}$, $\mu_*(x,y) = \mu^{k+1}(x,y)$.
5. Compute the optimal value of the goal function

$$\mathrm{P}(\varsigma) = \int_S \min_{l=\overline{1,m}} f(x,y) t_i(x,y)dxdy,$$

with $\varsigma = \varsigma_*$.

This concludes the algorithm description.

Efficiency of the partitioning of service areas can be estimated using the coefficient of uneven load on consumers [1]:

$$k_i = \frac{\int_{S_i} f(x,y)dxdy}{\min\left\{\int_{S_1} f(x,y)dxdy, ..., \int_{S_m} f(x,y)dxdy\right\}}, \; i = \overline{1,m}. \tag{15}$$

Here the value $\int_{S_i} f(x,y)dxdy$ denotes the number of consumers in the service area of the station ς_i, $i = \overline{1,m}$.

Conclusions and recommendations for further research. We have examined a particular problem of optimal coverage or partitioning of a set of consumers into service areas without restrictions on the power of service stations, locating the coordinates of subset centers. We have analyzed existing formulations of related problems and methods for their solution. We have constructed an economic-mathematical model of the problem, developed an algorithm for its solution. We proposed an efficiency criterion of the partition for service areas which is defined as the coefficient of uneven load on consumers.

For example, problems (2) or (3) can be applied to placement of post offices or stores selling essential goods, planning locations of pedestrian crossings, waste recycling facilities and other objects as well as determining the boundaries of these services. Solving such problems requires additional research, including collecting and statistically processing additional information about possible ways of consumer movements and service station locations. An analysis of the infrastructure of a populated area or region will allow to identify bottlenecks in the functioning of various organizations, elimination of which will lead to an increase in the quality of life of the population.

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Економіко-математична модель оптимального розміщення станцій обслуговування

У статті досліджена одна спеціального вигляду задача оптимального покриття або розбиття множини споживачів на зони обслуговування без обмежень на потужності станцій з відшуканням координат центрів підмножин, проведено аналіз існуючих постановок споріднених задач та методів їх розв’язання. Побудована економіко-математична модель задачі, розроблено алгоритм її розв’язання, запропоновано критерій ефективності розбиття на сегменти обслуговування, що визначається як коефіцієнт нерівномірності навантаження по споживачам. Наведено приклади практичних задач, для розв’язання яких можна застосувати розроблену модель та алгоритм. Для розв’язання таких задач необхідно проведення додаткових досліджень, зібрати та статистично обробити додаткову інформацію про можливі шляхи пересування споживачів та можливі місця розміщення станцій обслуговування. Аналіз інфраструктури населеного пункту або регіону дозволить з’ясувати вузькі проблемні місця в функціонуванні різних організацій, усунення яких призведе до підвищення якості життя населення.

Ключові слова: економіко-математична модель, оптимальне розбиття, станція обслуговування, алгоритм, критерій, ефективність розбиття, характеристична функція.

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Экономико-математическая модель оптимального размещения станций обслуживания

В статье исследована одна специального вида задача оптимального покрытия или разбиение множества потребителей на зоны обслуживания без ограничений на мощности станций с отысканием координат центров подмножеств, проведен анализ существующих постановок родственных задач и методов их решения. Построена экономико-математическая модель задачи, разработан алгоритм ее решения, предложен критерий эффективности разбиения на сегменты обслуживания, определяется как коэффициент неравномерности нагрузки по потребителям. Приведены примеры практических задач, для решения которых можно применить разработанную модель и алгоритм. Для решения таких задач необходимо проведение дополнительных исследований, собрать и статистически обработать дополнительную информацию о возможных путях передвижения потребителей и возможные места размещения станций обслуживания. Анализ инфраструктуры населенного пункта или региона позволит выяснить узкие проблемные места в функционировании различных организаций, устранение которых приведет к повышению качества жизни населения.

Ключевые слова: экономико-математическая модель, оптимальное разбиение, станция обслуживания, алгоритм, критерий, эффективность разбиения, характеристическая функция.